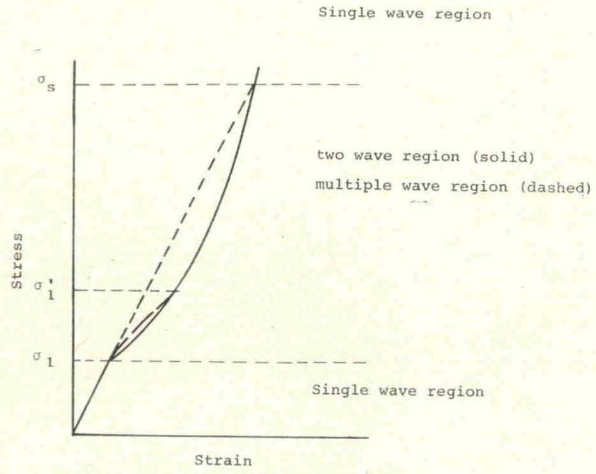

FIG. 3. Schematic diagram of possible response curves for shock compression.

In Ref. 8 conditions were discussed in which more than one shock wave are generated. Two shock waves are expected to be generated in a material which is shock compressed to a stress slightly above its dynamic elastic limit or phase transition pressure. In such cases the stress and strain behind the second wave are

$$\sigma_2=[\rho_0 U_{s1}(U_{s2}-U_{p1})(U_{p2}-U_{p1})/(U_{s1}-U_{p1})]+\sigma_1, \tag{3}$$

and

$$\epsilon_2=1-[(U_{s1}-U_{p1})(U_{s2}-U_{p2})/U_{s1}(U_{s2}-U_{p1})], \tag{4}$$

where U_{s1} and U_{s2} are the velocities in laboratory coordinates of the first and second shock waves, and U_{p1} and U_{p2} are the material velocities behind the first and second waves. The regions in which these equations are applicable are shown in the Hugoniot diagram of Fig. 3. The Hugoniot shown in Fig. 3 corresponds to a material with a yield point or phase transition at σ_1. If the transition or yielding does not occur at a well specified stress but occurs over a range of stresses (σ_1 to σ_1' in Fig. 3) a shock wave fan will be generated for stresses between σ_1 and σ_s. For this case Eqs. (3) and (4) may be generalized to n shock waves. Thus,

$$\sigma_n=\sigma_{n-1}+[\rho_0(U_{sn}-U_{p,n-1})(U_{pn}-U_{p,n-1})/(1-\epsilon_{n-1})] \tag{5}$$

and

$$\epsilon_n=1-\prod_{i=1}^{i=n}[(U_{si}-U_{p1})/(U_{s1}-U_{p,i-1})]. \tag{6}$$

For the wedge configuration used in this series of experiments, it is possible to determine the general features of the Hugoniot by analysis of the shape of the free surface. Thus in Fig. 2, it is seen that the free surface has two points of slope change which correspond to the intersection of two waves with the free surface. The analysis of these data are based on the two wave configuration shown in Fig. 4. For such a two-wave system, the first wave is the elastic wave and transmits a stress corresponding to the dynamic elastic limit or yield point. Upon its reflection at the free surface, two waves reflect into the sample, a dilatational and a shear wave. Because the second wave or plastic wave is well above the yield point, shear forces are not considered so that only a single longitudinal decompression wave is considered

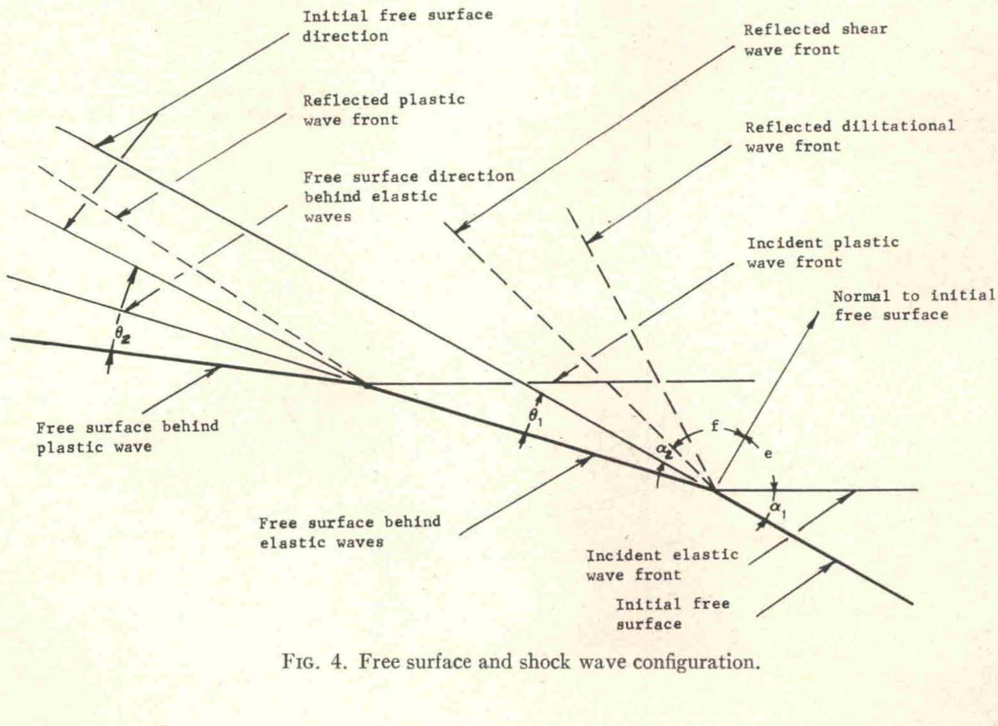

FIG. 4. Free surface and shock wave configuration.

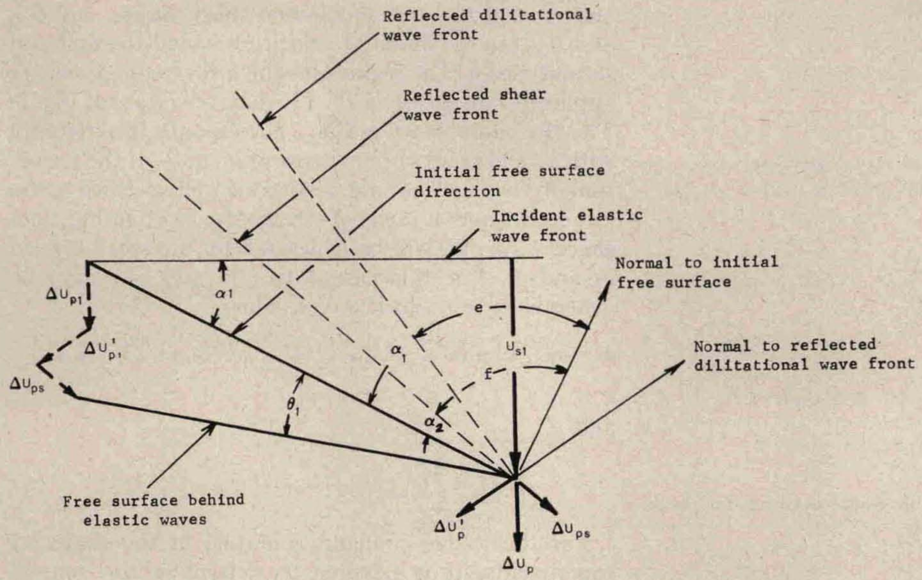

FIG. 5. Vector diagram for elastic wave interaction with the free surface.

to be reflected at the free surface. The vector diagrams in Figs. 5 and 6 show the material velocities associated with each of these waves and their relation to the free surface angles which may be measured in each experiment. In addition, in order to use Eqs. (3) and (4) to find the stress and strain behind each wave, it is necessary to determine the material velocities, U_{p1} and U_{p2}, which occur behind the first and second waves within the sample.

From Fig. 5, the free surface angle θ_1, is

$$\tan\theta_1 = \frac{(1+r_1)\cos\alpha_1 + r_2\sin\alpha_2}{1/\epsilon_1 - (1-r_1)\sin\alpha_1 - r_2\cos\alpha_2}, \qquad (7)$$

where

$$r_1 = \Delta U_{p1}'/\Delta U_{p1}$$

and

$$r_2 = \Delta U_{ps}/\Delta U_{p1}$$

are the reflected material velocity ratios for the dilatational and shear waves, respectively; α_1 and α_2 are the shock front angles between the free surface and the dilatational and shear wave fronts, respectively; ϵ_1 is the strain at the yield point as defined in Eq. (2); ΔU_{p1} is the material velocity occurring behind the incident elastic wave; and $\Delta U_{p1}'$ and ΔU_{ps} are the material velocities which occur behind the reflected dilatational and shear waves, respectively, as shown in Fig. 5.

The velocity ratios are related to the angles of obliquity e and f as shown in Fig. 5 by the relationships[9–11]

$$\frac{\Delta U_{p1}'}{\Delta U_{p1}} = \frac{4\tan f\tan e - (\tan^2 f - 1)g(\nu)}{4\tan f\tan e + (\tan^2 f - 1)g(\nu)} \qquad (8)$$

and

$$\frac{\Delta U_{ps}}{\Delta U_{p1}} = \frac{-4\tan e\, g(\nu)}{4\tan e\tan f + (\tan^2 f - 1)g(\nu)}, \qquad (9)$$

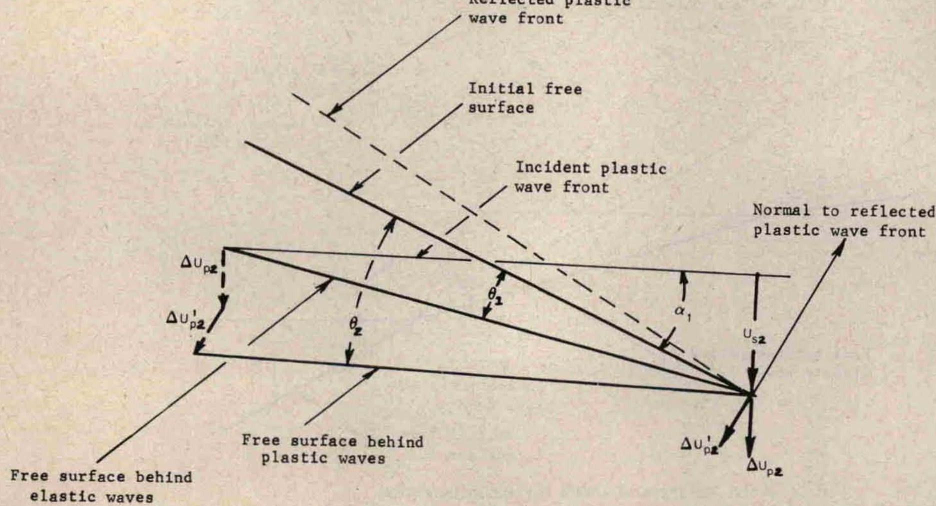

FIG. 6. Vector diagram for plastic wave interaction with the free surface.